# **Trigonometry and Modelling Cheat Sheet**

This chapter builds upon the previous, introducing more useful methods, formulae and identities relating to trigonometric functions

#### Addition Formulae

- $sin(A + B) \equiv sinAcosB + cosAsinB$   $sin(A B) \equiv sinAcosB cosAsinB$
- $cos(A + B) \equiv cosAcosB sinAsinB$   $cos(A B) \equiv cosAcosB + sinAsinB$
- $\tan(A+B) \equiv \frac{tanA + tanB}{1 tanAtanB} \qquad \qquad tan(A-B) \equiv \frac{tanA tanB}{1 + tanAtanB}$

You need to know how to use the above formulae to find exact values of trigonometric functions for various angles.

Example 1: Show, using the formula for 
$$\sin(A+B)$$
, that  $\sin(75^\circ) = \frac{\sqrt{6}+\sqrt{2}}{4}$ 

We can rewrite  $\sin(75^\circ)$  as  $\sin(45^\circ + 30^\circ)$ . We choose 45 and 30 because we know the exact values of  $\sin(45^\circ)$ ,  $\cos(45^\circ)$ ,  $\sin(30^\circ)$  and  $\cos(30^\circ)$ , so when we put them into the addition formula, we will have all terms given as exact values.

$$\sin(75^\circ) \equiv \sin(45+30)$$

$$\sin(45+30) \equiv \sin45\cos30 + \cos45\sin30$$

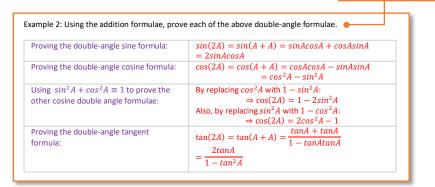
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

#### Double-angle formulae

- $sin(2A) \equiv 2sinAcosA$
- $cos(2A) \equiv cos^2 A sin^2 A = 1 2sin^2 A = 2cos^2 A 1$
- $\tan(2A) \equiv \frac{2tanA}{1 tan^2A}$

You can be asked to reproduce these proofs



You can see that there are three different versions for the cosine double angle formula. It is important you are familiar with all three as one may be more useful than the others in certain questions

Example 3: Simplify as much as possible the expression: 
$$sin^4x - 2sin^2xcos^2x + cos^4x$$

Spotting the factorisation:  $sin^4x - 2sin^2xcos^2x + cos^4x = (cos^2x - sin^2x)^2$ 

Using  $cos2x = cos^2x - sin^2x$ :  $= (cos2x)^2 = cos^22x$ 

Example 4: Simplify as much as possible the expression: 
$$\sqrt{1 + cosx}$$

Since  $cos2x = 2cos^2x - 1$ 

Substituting this result into the given expression: 
$$\Rightarrow \sqrt{1 + cosx} = \sqrt{1 + 2cos^2\left(\frac{x}{2}\right) - 1} = \sqrt{2cos^2\left(\frac{x}{2}\right)}$$



#### Simplifying $asinx \pm bcosx$

Expressions of the above form can be simplified into one trigonometric term.

- $asinx \pm bcosx$  can be expressed as  $Rsin(x \pm \alpha)$
- $acosx \pm bsinx$  can be expressed as  $Rcos(x \mp \alpha)$

When the coefficient of sin is positive, use  $Rsin(x\pm\alpha)$  and when the coefficient of cos is positive, use  $Rcos(x \mp \alpha)$ . Of course, when both coefficients are

where a, b, R > 0 and  $0 < \alpha < \frac{\pi}{a}$ .

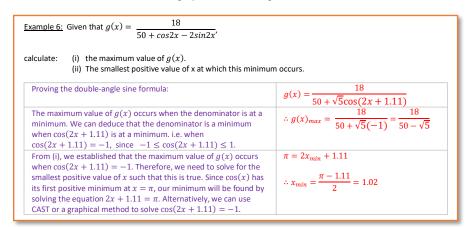
The procedure for achieving the above simplifications can be broken down into three steps:

- Expand the form using the addition formulae, and equate it to  $asinx \pm bcosx$
- [2] Compare the coefficients of sinx and cosx on both sides of the equation, to get two equations in terms of R and  $\alpha$ .
- [3] Solve these simultaneously to find R and  $\alpha$ .

Proving the double-angle sine formula:	$1\cos 2x - 2\sin 2x = R\cos(2x + \alpha) \equiv R\cos 2x \cos \alpha - R\sin 2x \sin \alpha$
Equating coefficients:	$1 = R\cos\alpha$ (1) (equating $\cos 2x$ coefficients) $2 = R\sin\alpha$ (2) (equating $\sin 2x$ coefficients)
Solving simultaneously.: We divide equation [2] by [1].	$tan\alpha = \frac{Rsin\alpha}{Rcos\alpha} = \frac{2}{1} = 2$ $\therefore \alpha = \arctan(2) = 1.11$
Finding $R$ : Square equations [1] and [2] then add them together. We also use the identity $cos^2\alpha + sin^2\alpha \equiv 1$	$(1)^2 + (2)^2 \Rightarrow R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = (1)^2 + (2)^2$ $\Rightarrow R^2 (\cos^2 \alpha + \sin^2 \alpha) = 5$ $\Rightarrow R^2 = 5 \therefore R = \sqrt{5}$
Putting everything together:	So $\cos 2x - 2\sin 2x = \sqrt{5}\cos(2x + 1.11)$

A shortcut for finding R is to use  $R = \sqrt{a^2 + b^2}$ 

This form is often useful because it makes solving equations and finding minimum/maximum values much easier



#### Solving equations

To solve more complicated trigonometric expressions, you will first need to simplify the equation using the formulae and methods we have covered so far. Here is an example showing how we do this in practice:

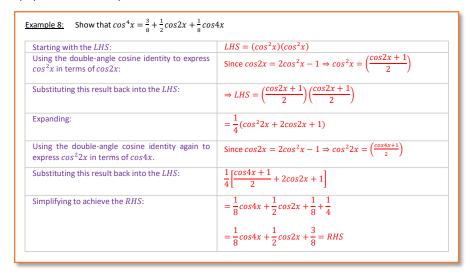
Using the addition formulae	3sinxcos45 - 3cosxsin45 - sinxcos45 - cosxsin45 =
Simplifying	2sinxcos45 - 4cosxsin45 = 0
Using exact values for $cos45^{\circ}$ , $sin45^{\circ}$	$\sqrt{2}sinx - 2\sqrt{2}cosx = 0$
Dividing through by $\sqrt{2}$	sinx = 2cosx
Dividing through by <i>cosx</i> and finding the principal solution:	$tanx = 2 : x = \arctan(2) = 63.4^{\circ}$
Using CAST or a graphical method, we can find all the solutions:	The solutions in the given interval are: $x = 63.4^{\circ}.243.4^{\circ}.$

## Pure Year 2

#### **Proving identities**

You need to be able to use everything we have covered so far to prove identities. You must start from one side of the equation and use your knowledge of trigonometric identities to manipulate the expression and achieve what is on the other side.

There is no set procedure to follow in your manipulation. Your knowledge of the identities is being tested, so you need to make sure you are very familiar with the content in this chapter and the previous. As with most of Mathematics, the mos useful preparation tool here is practice.



### Modelling with trigonometric functions

In the exam you will likely be given problems where trigonometric functions are used to model real-life situations, ofter involving the forms  $Rsin(x\pm\alpha)$  and  $Rcos(x\pm\alpha)$ . To succeed in these questions, you must properly understand the scenario given to you. Read through the text more than once to make sure you understand what is going on. The maths itself is the same as before; you just need to be able to apply it in the context of the question.

